# Math 409 Midterm 2 practice #1

**Name**: ______________________

**This exam has 4 questions, for a total of 100 points.**

Please answer each question in the space provided. No aids are permitted.

**Question 1. (40 pts)**

In each of the following eight cases, indicate whether the given statement is true or false. No justification is necessary.

(a) Let $\{x_n\}$ and $\{y_n\}$ be convergent sequences in $\mathbb{R}$. Then the sequence $\{x_n y_n\}$ converges.

(b) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x-1}$ does not exist.

(c) Let $\{x_n\}$ be a sequence such that $x_n \in (0,1)$ for every $n \in \mathbb{N}$. Then $\{x_n\}$ has a subsequence which is Cauchy.

(d) $\lim_{x\to\infty} \frac{x - 2x^2 + 5x^3}{6 - x + x^2} = \infty$

(e) Let $\{x_n\}$ be a sequence in $\mathbb{R}$ with the property that each of its subsequences has a convergent subsequence. Then $\{x_n\}$ is bounded.

(f) If a function is differentiable on $\mathbb{R}$, then it is uniformly continuous on $\mathbb{R}$.

(g) Let $f$ be a function which is uniformly continuous on $\mathbb{R}$. Then the function $g$ defined by $g(x) = f(f(x))$ for all $x \in \mathbb{R}$ is uniformly continuous on $\mathbb{R}$.

(h) If $f\colon (0,1) \to \mathbb{R}$ is continuous and bounded, then $f$ is uniformly continuous.

**Question 2. (20 pts)**

(a) Let $f$ be a function defined on an open interval containing a given point $a$. State what it means for $f(x)$ to converge to a number $L$ as $x$ approaches $a$.

(b) Let $a \in \mathbb{R}$ and let $f$ and $g$ be functions on $\mathbb{R}$ such that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ both exist. Prove directly from the definition of a limit that $\lim_{x \to a}(f+g)(x)$ exists.

**Question 3. (20 pts)**

(a) State the Extreme Value Theorem.

(b) Give an example of a function $f$ which is bounded on $[0, 1]$ but does not have a maximum on $[0, 1]$.

**Question 4. (20 pts)**

(a) State the Intermediate Value Theorem.

(b) Assuming the fact that the function $\cos x$ is continuous on $\mathbb{R}$, prove that there exists an $x \in \mathbb{R}$ such that $x^6 + x^4 + 1 = 2\cos x^3$.